

Lorentz violation in gravity

Diego Blas

CERN, Theory Division, 1211 Geneva, Switzerland.

The study of gravitational theories without Lorentz invariance plays an important role to understand different aspects of gravitation. In this short contribution we will describe the construction, main advantages and some phenomenological considerations associated with the presence of a preferred time direction.

1 Introduction

One hundred years after its formulation, General Relativity (GR) is living a golden era of continuous verifications of its predictions, at many scales and in very different processes^{1,2,3}. The agreement of data with GR predictions is both astonishing (given the range of scales probed) and frustrating (since GR can not be a complete quantum theory, but we lack experimental guidance towards its completion)^a.

Besides confirming the predictions of GR, the current data can also be used to constrain possible deviations. This is an important program from which we can learn about the robustness of the different properties of GR, the benefits of modifying them and the viability of the resulting alternative theories. This brief note is devoted to a particular modification suggested by theories of gravity with a better quantum behaviour than GR. To attain this, they abandon one of the principles of GR: Lorentz invariance. We will only discuss the case of a preferred frame defining a time direction at every point of space-time. After introducing the formalism and explaining its possible relation to quantum gravity, we will proceed to derive some of its phenomenological consequences. For further information, the reader can consult the recent review paper⁴.

2 Theoretical construction

We will consider metric theories where there is a local preferred time direction represented by a time-like vector field u^μ satisfying

$$u_\mu u^\mu = 1. \tag{1}$$

If this vector is otherwise generic, these theories are known as Einstein-aether theories⁵. Their relation to quantum gravity is not completely clear, but their study may be important for fundamental theories of gravity where Lorentz invariance is broken (spontaneously or fundamentally) by the selection of a preferred frame. A more concrete example of how this may happen is provided by Hořava gravity, which assumes the existence of a preferred foliation of space-time into space-like hypersurfaces⁶. This allows to construct a theory of gravity renormalizable by power-counting and close to GR at low-energies. In terms of the vector satisfying (1), the theory

^aThe existence of dark matter and dark energy is sometimes considered as a hint towards the construction of alternatives to GR. This motivation is certainly valid though the standard paradigm based on GR is consistent both theoretically and phenomenologically.

requires the existence of a field φ representing the foliation and

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi}}. \quad (2)$$

The generic theories defined with a vector of the form (2) are known as khronometric theories⁷. To construct the action we write the different operators including u^μ and $g^{\mu\nu}$, covariant under diffeomorphisms and organized in a derivative expansion (we also assume CPT),

$$S = -\frac{M_0^2}{2} \int d^4x \sqrt{-g} \left(R + K^{\alpha\beta}_{\mu\nu} \nabla_\alpha u^\mu \nabla_\beta u^\nu + \lambda(u^\mu u_\mu - 1) + \frac{\mathcal{O}^{n+2}}{M_\star^n} \right), \quad (3)$$

where g and R are the metric determinant and the Ricci scalar and

$$K^{\alpha\beta}_{\mu\nu} \equiv c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta_\mu^\alpha \delta_\nu^\beta + c_3 \delta_\nu^\alpha \delta_\mu^\beta + c_4 u^\alpha u^\beta g_{\mu\nu}, \quad (4)$$

We use the constant M_0 instead of M_P for the mass scale in front of the Einstein-Hilbert action to distinguish it from the quantity appearing in Newton's law⁴. By the last term in (3) we indicate the higher dimensional operators, which we assume to be suppressed by a common scale M_\star . We imposed the restriction (1) through a Lagrange multiplier λ . In the khronometric case, this is not necessary. Furthermore, the condition (2) implies that one of the terms in (4) can be expressed in terms of the others. One then absorbs the c_1 term into the other three terms by multiplying the second, third and forth term respectively by the new couplings

$$\lambda \equiv c_2, \quad \beta \equiv c_3 + c_1, \quad \alpha \equiv c_4 + c_1. \quad (5)$$

For a reformulation of (3) in terms of geometrical quantities of the congruences of u^μ see⁸.

3 Short distance modifications

Let us first discuss the operators of (3) suppressed by the scale M_\star . Since we suppose that Lorentz invariance is broken in a regime where gravity is weakly coupled, one can start parameterizing the changes in gravitation by considering linear equations for the perturbations of the metric with Lorentz violating (LV) terms. Assuming that parity and time reversal are not violated and that the equations are at most second order in time derivatives, we can introduce the dispersion relations

$$\omega^2 = p^2 \left(1 + \sum_{n=1}^L \alpha_n \left(\frac{p}{M_\star^{gw}} \right)^{2n} \right), \quad (6)$$

for the propagating degrees of freedom (e.g the graviton) and the modified Poisson's equation

$$p^2 \left(1 + \sum_{n=1}^L \beta_n \left(\frac{p}{M_\star^\phi} \right)^{2n} \right) \phi = -\frac{1}{2M_0^2} \tau_{00}, \quad (7)$$

for the potentials ϕ sourced by matter's energy, represented by τ_{00} . The mass scales M_\star^ϕ and M_\star^{gw} are kept independent, even if they are both related to M_\star .

Let us first discuss the modifications of the graviton's dispersion relation, Eq. (6). If the gravitational waves (GW) have the dispersion relation (6), this modifies the frequency dependence in the propagation of the wave-fronts, which may be observed by future detectors of GW. These effects will be very suppressed if we assume that $M_\star^{gw} \approx M_\star^\phi$, given that the latter are constrained to be $M_\star^\phi < (\mu m)^{-1} \approx 10^{-2}$ eV (see below). They may still have an impact for the GWs generated in the primordial universe since in this case the typical energies during production may be almost as high as M_P . Thus, if primordial GWs are observed, the range of energies at which LV is tested (in fact any short-distance modification) will improve dramatically.

More is known about possible deviations of the potentials at high-energies, Eq. (7). Let us focus on the case relevant for the short distance behaviour of Hořava gravity where only $\beta_2 \neq 0$, and absorb its value into M_\star^ϕ ($M_\star^\phi \mapsto M_\star^\phi \beta_2^{1/4} 2^{1/8}$). Taking a point particle of mass m_{pp} at rest as a source, the solution of Eq. (7) away from the source is

$$\phi = -\frac{m_{pp}}{8\pi M_0^2 r} \left[1 - e^{-M_\star^\phi r} \cos(M_\star^\phi r) \right], \quad (8)$$

where r is the distance from the source. This potential regularises the divergent behaviour of Newton's potential at small distances. Furthermore, at scales where the deviations start to be important, it is similar to the potentials that have been considered to constrain the deviations from Newton's law at short distances¹⁰

$$\phi = -\frac{m_{pp}}{8\pi M_0^2 r} \left[1 + \tilde{\alpha} e^{-r/\tilde{\lambda}} \right]. \quad (9)$$

From these works, one concludes that a bound of the form $M_\star^\phi \lesssim (\mu m)^{-1}$ should apply. However, the differences between the potentials of Eq. (9) and (8) are important, e.g. the potentials in (9) are singular at short distances except for $\tilde{\alpha} = -1$ and they do not present the oscillatory behaviour of (8). Thus, a precise bound on M_\star^ϕ requires a reanalysis of the experimental data.

Finally, even though our previous discussion was organised around the modified equations (6) and (7), the effects of LV at short distances (high energies) may also be important for the background evolution in the primordial universe, see⁴ for the relevant references.

4 Long distance modifications

By long distance modifications we mean the theory whose gravitational action is given by (3) in the limit $M_\star \rightarrow \infty$. Independently of the coupling to matter, some bounds can be derived on the constants c_i from stability of perturbations around a Minkowski background and requiring the absence of gravitational Cherenkov radiation⁴. To find the phenomenological predictions of the theory we first need to understand how u^μ and $g_{\mu\nu}$ couple to matter. In principle, any coupling should be allowed. The generic consequence would be the presence of big deviations from Lorentz invariance also in the standard model of particle physics. These deviations are extremely small, cf. ¹¹, which makes it natural to assume that, as long as gravitational test are concerned, matter is not coupled to u^μ at all. Explaining how this can happen in a natural way while keeping the other couplings to u^μ not too small remains an open challenge for the set-up. Different possibilities have been explored⁴, but no definite mechanism has been produced yet. Notice that for dark matter and dark energy one can keep these couplings arbitrary.

Bounds on the LV parameters in (3) come from different observations. Assuming that the preferred frame u^μ is aligned with the CMB, which is a reasonable supposition^{1,4}, one finds that the gravitational potential in the Solar System is modified by the presence of two post-Newtonian parameters α_1 and α_2 . These two parameters are functions of the LV parameters. For instance, for the khronometric parameters (5) they read

$$\alpha_1 = -4(\alpha - 2\beta), \quad \alpha_2 = \frac{(\alpha - 2\beta)(\alpha - \lambda - 3\beta)}{2(\lambda + \beta)}. \quad (10)$$

Current observations yield the bounds¹ $|\alpha_1| < 10^{-4}$ and $|\alpha_2| < 10^{-7}$.

Further bounds can be obtained from strongly gravitating bodies. In this case, even if matter is not coupled to u^μ , the gravitons in the body are, and for objects with large gravitational fields (very compact) the body as a whole will *effectively* feel the presence of u^μ . To parametrize this for the phenomena at large distances with respect to the size of the source, one can assume the action of the point particle to be

$$S_{pp \ A} = - \int ds_A \tilde{m}_A(\gamma_A), \quad (11)$$

where \tilde{m}_A is a function of $\gamma_A \equiv u_\mu v_A^\mu$ and v_A^μ is the four-velocity of the source (see¹² for similar descriptions in scalar-tensor theories). Finally, ds_A is the line element of the trajectory. If $\gamma_A \ll 1$, one can expand the action to second order in γ_A and describe the physics in terms of

$$\tilde{m}_A|_{\gamma_A=1}, \quad \sigma_A \equiv - \left. \frac{d \ln \tilde{m}_A(\gamma_A)}{d \ln \gamma_A} \right|_{\gamma_A=1}. \quad (12)$$

The parameters σ_A are called *sensitivities* and represent the effective coupling of the source to u^μ . The presence of these couplings introduces an extra force in the dynamics of binary systems, which precludes the conservation of the usual momentum sourcing gravitational waves^b. This implies the emission of dipolar radiation, which is absent in GR. Since the latter is enhanced with respect to the quadrupolar radiation by a factors $(c/v)^2$, where v is the typical orbital velocity of the system, this means that even for very small σ_A (corresponding to neutron stars), one can get very tight bounds on the LV parameter space by observing the radiation damping of binary pulsars⁹. The same is also true for solitary pulsars, where the bounds come from changes in the spin-precession. For these bounds to relate to the fundamental parameters in (3) one needs to compute the numbers σ_A for different sources, which was done in⁹.

Finally, cosmological observations produce further bounds on the LV parameters. Remarkably, these also apply to the possible LV in the dark matter. The bounds come from different observations: first, the Friedmann equation is modified by a renormalization of Newton's constant depending on the LV parameters. This deviation can be constrained with the data from big bang nucleosynthesis, by the growth of structure (controlled by the local G_N) and CMB observations¹³. Furthermore, the existence of the vector field u^μ introduces a source of anisotropic stress present at many scales, and which can be constrained by CMB observations. Similarly, the possible coupling of dark matter to u^μ introduces an extra force in dark matter, which may violate the weak equivalence principle. This has consequences for the gravitational dynamics of dark matter¹³. The cosmological observations from the regimes where linearized cosmology is applicable imply bounds close to the percent level for all the previous couplings¹³. These bounds will improve once the consequences for non-linear scales (scales below 10 Mpc) are understood.

References

1. C. M. Will, arXiv:1409.7871 [gr-qc].
2. E. Berti, E. Barausse, V. Cardoso, L. Gualtieri, P. Pani, U. Sperhake, L. C. Stein and N. Wex *et al.*, arXiv:1501.07274 [gr-qc].
3. T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. **513** (2012) 1
4. D. Blas and E. Lim, Int. J. Mod. Phys. D **23** (2015) 13, 1443009
5. T. Jacobson and D. Mattingly, Phys. Rev. D **64** (2001) 024028
6. P. Horava, Phys. Rev. D **79** (2009) 084008
7. D. Blas, O. Pujolas and S. Sibiryakov, JHEP **1104** (2011) 018
8. T. Jacobson, Phys. Rev. D **89** (2014) 8, 081501
9. K. Yagi, D. Blas, E. Barausse and N. Yunes, Phys. Rev. D **89** (2014) 084067
10. D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. **98** (2007) 021101
11. S. Liberati, Class. Quant. Grav. **30** (2013) 133001
12. T. Damour and G. Esposito-Farese, Class. Quant. Grav. **9** (1992) 2093.
13. B. Audren, D. Blas, M. M. Ivanov, J. Lesgourgues and S. Sibiryakov, JCAP **1503** (2015) 03, 016

^bThere is still a conserved momentum associated with translation invariance, but it differs from the one of GR.